Math 10B with Professor Stankova
Quiz 9; Tuesday, 4/2/2019
Section #206; Time: 9:30 AM
GSI name: Roy Zhao **Name:** ____________________

Circle True or False or leave blank. (1 point for correct answer, -1 for incorrect answer, 0 if left blank)

1. **TRUE** False It is possible to determine the equation for the PDF, given the equation for the CDF.

> **Solution:** Taking the derivative of the CDF gives the PDF.

2. True **FALSE** Suppose that $f(x) = x$ for $-0.5 \le x \le 1.5$ and 0 everywhere else. Then f is a PDF.

> **Solution:** This is false since $f(-0.5) = -0.5$ which is negative and PDFs cannot be negative.

Show your work and justify your answers. Please circle or box your final answer.

3. (10 points) (a) (2 points) Suppose that $f(x) = Cx^2$ for $-2 \le x \le 0$ and $f(x) = 0$ for all other x for some constant C. If f is a PDF, then find C.

> **Solution:** Since f is a PDF, we require that
> $$\int_{-\infty}^{\infty} f(x)dx = \int_{-2}^{0} Cx^2 = 1.$$
> This integral is
> $$\int_{-2}^{0} Cx = \frac{Cx^3}{3}|_{-2}^{0} = \frac{8C}{3} = 1.$$
> Therefore $C = \frac{3}{8}$.

(b) (4 points) Find the CDF of f from above. (Hint: the CDF will be piecewise)

> **Solution:** For $x \le -2$, then the CDF is 0 because the PDF is 0 there. Then for $-2 \le x \le 0$, we have that the CDF is
> $$F(x) = \int_{-\infty}^{x} f(t)dt = \int_{-2}^{x} f(t)dt = \frac{t^3}{8}|_{-2}^{x} = 1 + \frac{x^3}{8}.$$

So

$$F(x) = \begin{cases} 0 & x \leq -2 \\ 1 + \frac{x^3}{8} & -2 \leq x \leq 0 \\ 1 & x \geq 0. \end{cases}$$

(c) (4 points) Find the mean and median of the PDF f from above.

Solution: The mean is

$$\int_{-2}^{0} x \frac{3x^2}{8} dx = \int_{-2}^{0} \frac{3x^3}{8} = \frac{3x^4}{32}|_{-2}^{0} = \frac{-3}{2}.$$

The median is when the CDF is $\frac{1}{2}$ which is when $1 + \frac{x^3}{8} = \frac{1}{2}$ or at $x = -\sqrt[3]{4}$.